Pattern Avoidability with Involution

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Generalization: functional dependencies between variables

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$$x\,f(x)\,y$$

where f is any mapping.

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Generalization: functional dependencies between variables

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We consider **involutions** here.

Notation

- involution $\vartheta \circ \vartheta = \mathrm{id}$
  - morphic $\vartheta(uv) = \vartheta(u)\vartheta(v)$
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- instance w of pattern p for involution ϑ, if
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Example

011001 is an instance of $x\bar{x}x$

for morphic ϑ with $\vartheta(0) = 1,\ \vartheta(1) = 0$ where $h(x) = 01$.

Avoidability with Involution

Observation

For all patterns p with both x and $\bar{x}$ there exists an infinite word $\mathbf{w}$ such that no instance of p for an involution $\vartheta \neq \mathrm{id}$ occurs in $\mathbf{w}$.

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- pattern p is morphically (antimorphically) k-avoidable, if
  - there exists $\mathbf{w} \in A_k^\omega$ such that
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  minimal k such that p is morphically k-avoidable

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- morphic avoidance index $\mathcal{V}_m(p)$ of p:
  minimal k such that p is morphically k-avoidable
- $\mathcal{V}_m(p) = \infty$ if p unavoidable
- analogously, antimorphic avoidance index $\mathcal{V}_a(p)$ of p

Some Facts

Lemma

$$\mathcal{V}_m(x\bar{x}x) > 2 \quad \textit{and} \quad \mathcal{V}_a(x\bar{x}x) > 2.$$

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- $000, 111, 010, 101$ do not occur in $\mathbf{w}$

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- then 001100 has to occur in $\mathbf{w}$

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Suppose $\mathcal{V}_m(x\bar{x}x) = 2$ or $\mathcal{V}_a(x\bar{x}x) = 2$ with witness $\mathbf{w} \in A_2^\omega$,

- 000, 111, 010, 101 do not occur in $\mathbf{w}$
- then 001100 has to occur in $\mathbf{w}$
- but $00\vartheta(00)00 = 001100$ for ϑ with $\vartheta(0) = 1$ and $\vartheta(1) = 0$

Result

Theorem

$$\mathcal{V}_m(x\bar{x}x) = 3 \qquad \textit{and} \qquad \mathcal{V}_a(x\bar{x}x) = 3.$$

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- Thue-Morse word $\mathbf{v}$

  $\mathbf{v} = 0\,1\,1\,0\,1\,0\,0\,1\,1\,0\ \ldots$

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Consider morphic case.

- Thue-Morse word with subst. $\mathbf{w} = \mathbf{v}[0 \mapsto 0021,\ 1 \mapsto 0221]$

  $\mathbf{v} = 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ \ldots$

  $\mathbf{w} = 0021\ 0221\ 0221\ 0021\ 0221\ 0021\ 0021\ 0221\ 0221\ 0021\ \ldots$

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- let $|h(x)| \geq 7$ (smaller instance individually checked)

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- w.l.o.g. 0021 occurs in $h(x)$, and hence, $\vartheta(002)$ occurs in $\mathbf{w}$

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- then $\vartheta(002) = 002$ or $\vartheta(002) = 221$

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- then $\vartheta(002) = 002$ ~~or $\vartheta(002) = 221$~~ ; moreover $|h(x)| = 4n$

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- then $\vartheta(002) = 002$ ~~or $\vartheta(002) = 221$~~ ; moreover $|h(x)| = 4n$
- if 021 or 221 prefix of $h(x)$, then $0h(x\bar{x}x)$ implies cube in $\mathbf{v}$

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- if 021 or 221 prefix of $h(x)$, then $0h(x\bar{x}x)$ implies cube in $\mathbf{v}$
- if 00 or 02 suffix of $h(x)$, then $h(x\bar{x}x)21$ implies cube in $\mathbf{v}$

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- then $\vartheta(002) = 002$ ~~or $\vartheta(002) = 221$~~ ; moreover $|h(x)| = 4n$
- if 021 or 221 prefix of $h(x)$, then $0h(x\bar{x}x)$ implies cube in $\mathbf{v}$
- if 00 or 02 suffix of $h(x)$, then $h(x\bar{x}x)21$ implies cube in $\mathbf{v}$
- contradiction (antimorphic case similar)

Stripped Patterns

Lemma

Let $p \in (X \cup \{\bar{x}\})^*$ contain both x and $\bar{x}$. Then

$$\mathcal{V}(p[\bar{x} \mapsto x]) \leq \mathcal{V}_m(p) \leq \mathcal{V}(p[\bar{x} \mapsto y])$$

$$\mathcal{V}_a(p) \leq \mathcal{V}(p[\bar{x} \mapsto y])$$

with y new in p.

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Let $p \in (X \cup \{\bar{x}\})^*$ *contain both* x *and* $\bar{x}$*. Then*

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with y *new in* p*.*

Example

$\mathcal{V}(xxx) = 2$ and $\mathcal{V}_m(x\bar{x}x) = 3$ and $\mathcal{V}(xyx) = \infty$

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$$\mathcal{V}(xxx) = 2 \quad \leq \quad \mathcal{V}_m(xx\bar{x}) \quad \leq \quad \mathcal{V}(xxy) = 3$$

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- Suppose, $\mathcal{V}_m(xx\bar{x}) = 2$ with witness $\mathbf{w} \in A_2^\omega$,

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- both 00 and 11 do not occur in $\mathbf{w}$, that is, $\mathbf{w} = (01)^\omega$
- and $0101\vartheta(01)$ occurs in $\mathbf{w}$ for $\vartheta = \mathrm{id}$ (morphic) or $\vartheta(0) = 1$ and $\vartheta(1) = 0$ (antimorphic); contradiction

Complementary Patterns

Observation

Let p and p' be such that p' is p with x and $\bar{x}$ switched for all variables x. Then

$$\mathcal{V}_m(p) = \mathcal{V}_m(p') \qquad \textit{and} \qquad \mathcal{V}_a(p) = \mathcal{V}_a(p') \; .$$

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In particular

$$\mathcal{V}_m(\bar{x}x\bar{x}) = \mathcal{V}_a(\bar{x}x\bar{x}) = \mathcal{V}_m(\bar{x}\bar{x}x) = \mathcal{V}_a(\bar{x}\bar{x}x) = 3$$

In Summary

Theorem

Let $p \in \{x, \bar{x}\}^3$*, then*

$$\mathcal{V}_m(p) = \mathcal{V}_a(p) = \begin{cases} 3 & \textit{if } p \in \{x, \bar{x}\}^3 \setminus \{xxx, \bar{x}\bar{x}\bar{x}\} \\ 2 & \textit{otherwise}. \end{cases}$$

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Theorem (James Currie)

Let $p \in \{x, \bar{x}\}^*$ *and* $|p| \geq 4$*, then* $\mathcal{V}_m(p) = \mathcal{V}_a(p) = 2$*.*

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Corollary

Let $p \in \{x, \bar{x}\}^*$*, then*

$$\mathcal{V}_m(p) = \mathcal{V}_a(p) = \begin{cases} \infty & \text{if } p \in \{x, \bar{x}, x\bar{x}, \bar{x}x\} \\ 3 & \text{if } p \in \{x, \bar{x}\}^3 \setminus \{xxx, \bar{x}\bar{x}\bar{x}\} \\ 2 & \text{otherwise}. \end{cases}$$

— End of Talk —