Bounded Parikh Automata

M. Cadilhac[1], A. Finkel[2], and P. McKenzie[1]

[1]: Université de Montréal

[2]: ENS CACHAN

September 15th, 2011

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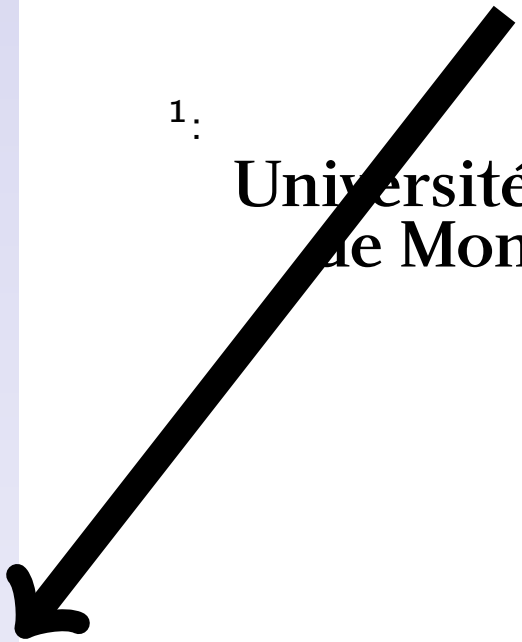

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Context

A practical problem:

- In model checking, properties described by automata

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- Small complexity classes (within NC^2)
- Word logics (variants of MSO)
- Algebra (pseudovarieties of monoids associated with language varieties)

Outline

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The result

Theorem

Parikh automata and their deterministic variant recognize the same bounded languages: those with a semilinear iteration set

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Definitions

- *Linear* set: of the form $E = \{\vec{c}_0 + \sum_{i=1}^{m} \vec{c}_i \cdot k_i \mid k_i \in \mathbb{N}\}$

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- *Semilinear* set: finite union of linear sets (equiv. FO[+])

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Why are they natural? One of many reasons:

Theorem ([Parikh, 1966])

With $\Sigma = \{a_1, \ldots, a_n\}$ *and* $w \in \Sigma^*$, *let*
$\text{Parikh}(w) = (|w|_{a_1}, \ldots, |w|_{a_n}) \in \mathbb{N}^n$ *be the Parikh image of* w.

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L *context-free* $\Rightarrow$ $\text{Parikh}(L)$ *semilinear*

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$$L = \{w \cdot a^{|w|+1} \mid w \in \{a, b\}^*\} \in \text{PA}$$

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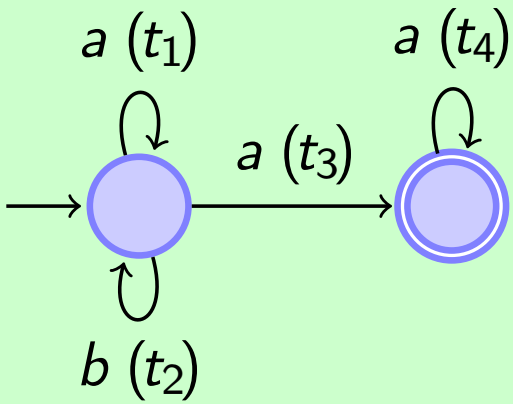

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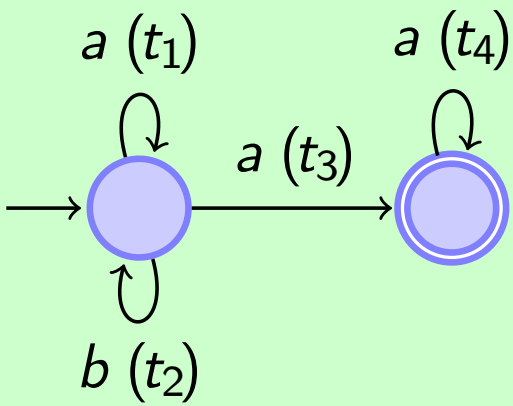

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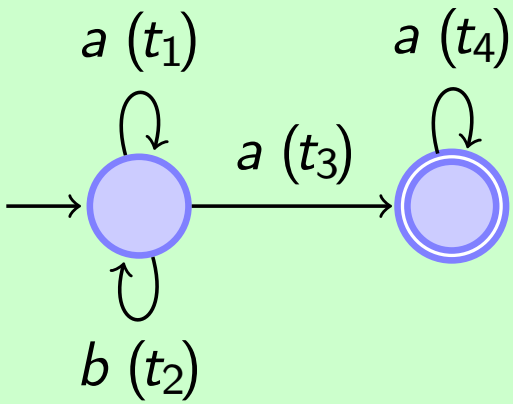

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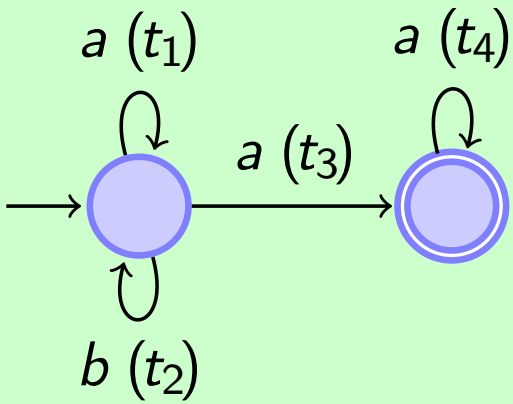

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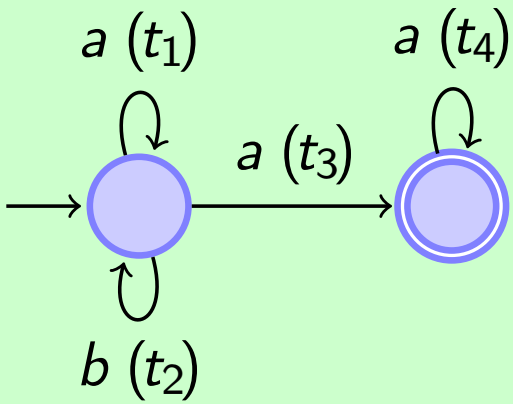

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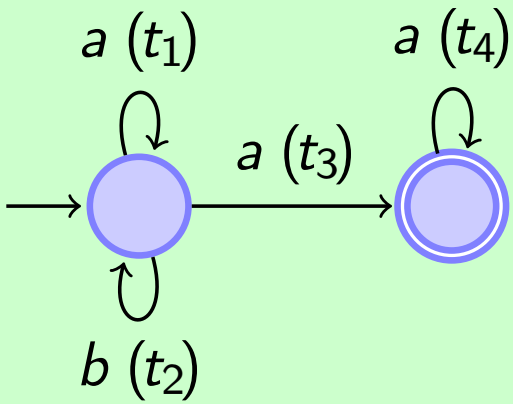

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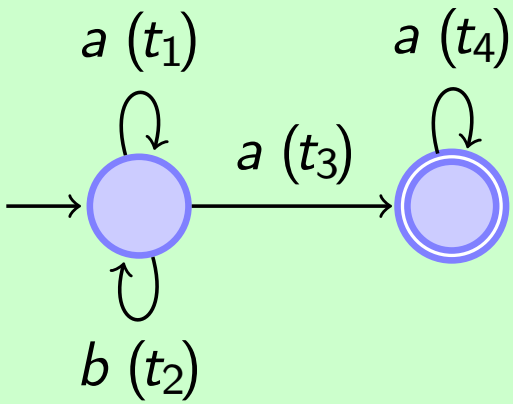

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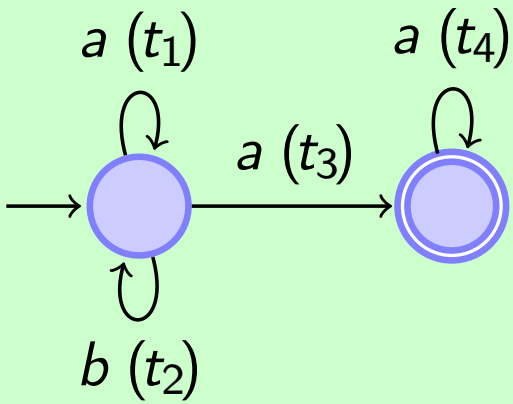

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Parikh(traced run):
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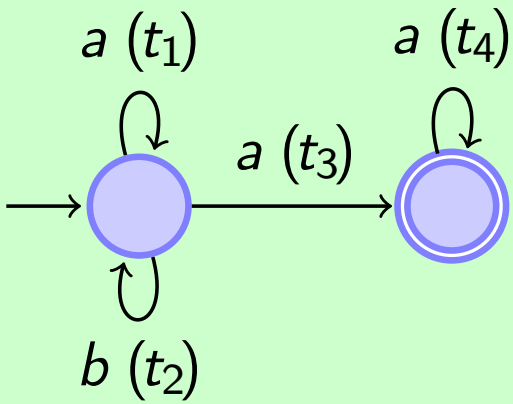

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Parikh(traced run): (0, 0, 0, 0)
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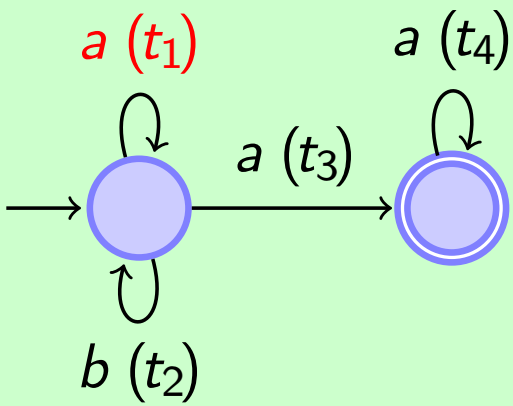

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Parikh(traced run): (1, 0, 0, 0)
$\pi = t_1$

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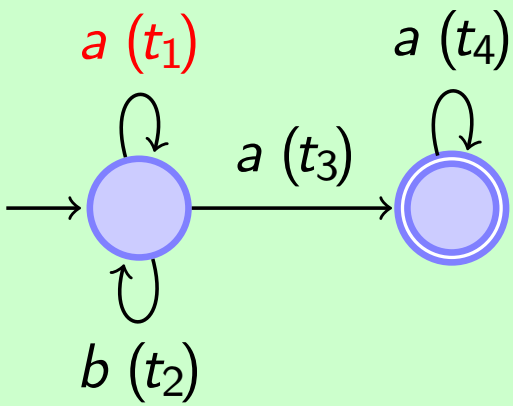

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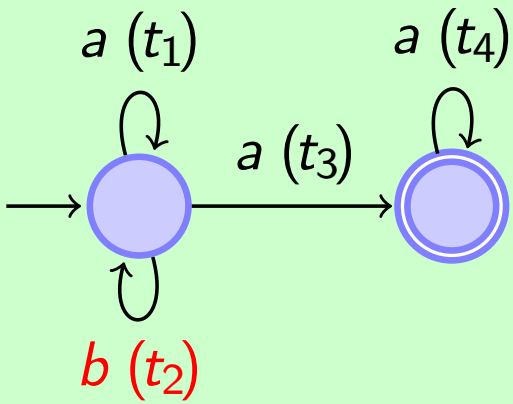

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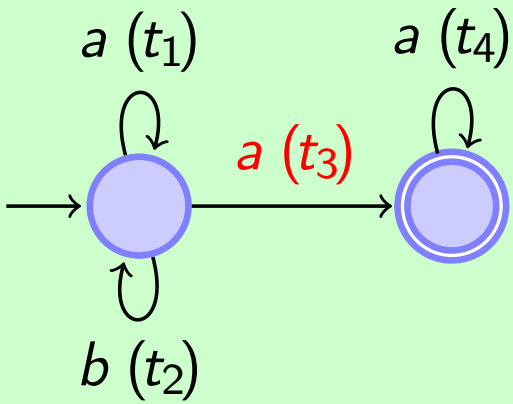

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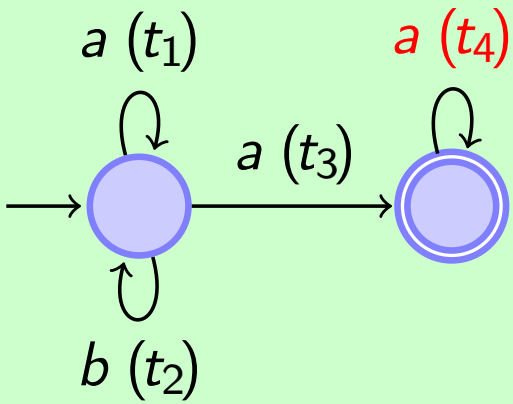

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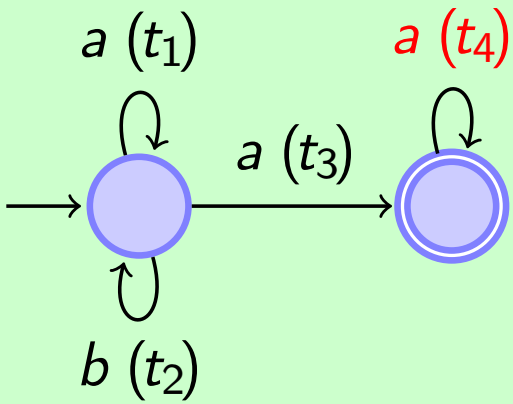

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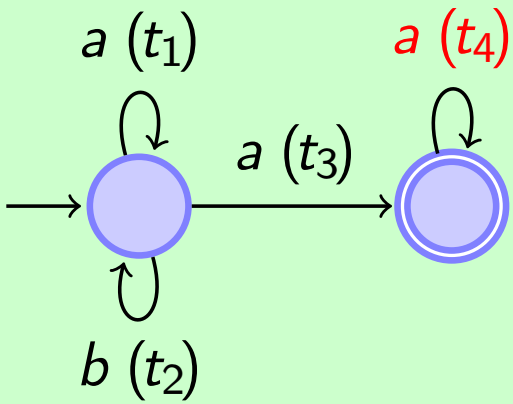

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Parikh(traced run): (2, 1, 1, 3)
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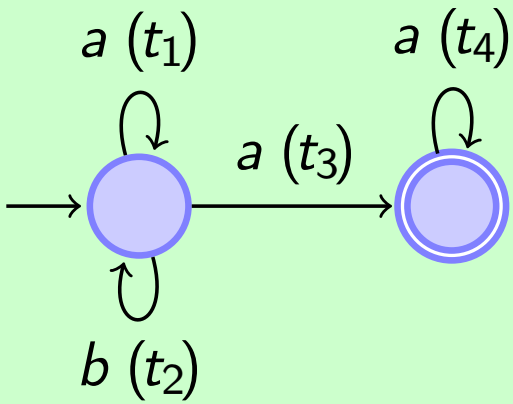

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Parikh(traced run): $(2, 1, 1, 3) \in C$

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Definition ([Klaedtke and Rueß, 2003])

- *Parikh automaton* (PA): a pair (A, C) with:
  - A a finite automaton of transition set δ
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Definition: Parikh automata

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- BSL intensively studied, e.g., Ginsburg & Spanier, 60's

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The big picture

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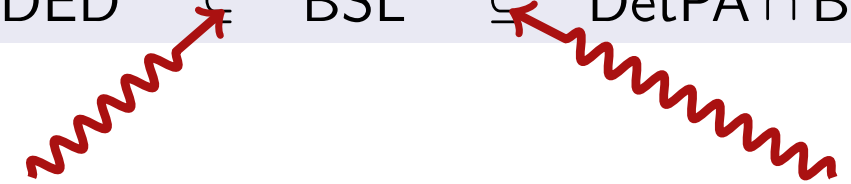

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Rest of this talk

Outline

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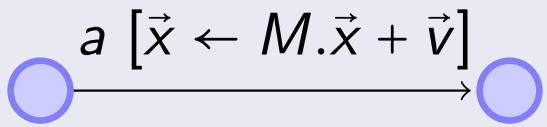

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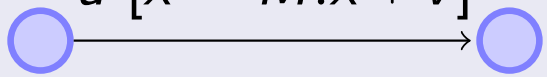

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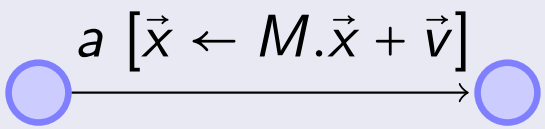

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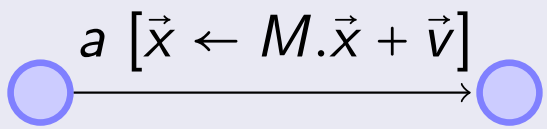

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- PA: APA in which every M = Identity
- Known facts: PAL, COPY, $\{a^{2^n}\} \in$ APA $\setminus$ PA
- Open: Dyck $\notin$ APA, PAL $\notin$ DetAPA

Outline

Result and definitions

BSL ⊆ DetPA

BSL ⊆ DetAPA with some property X

DetAPA with this property ⊆ DetPA

Corollaries and Further Work

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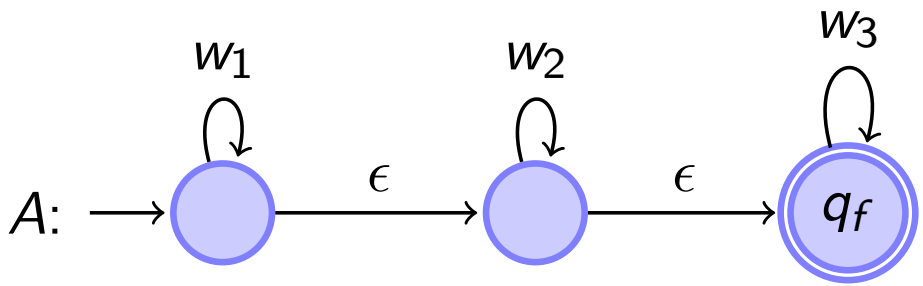
- Let $L \subseteq w_1^* w_2^* w_3^* \in$ BSL. Then a PA describes L:



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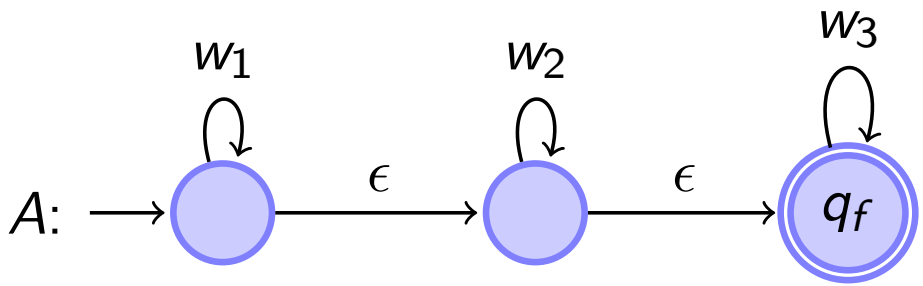

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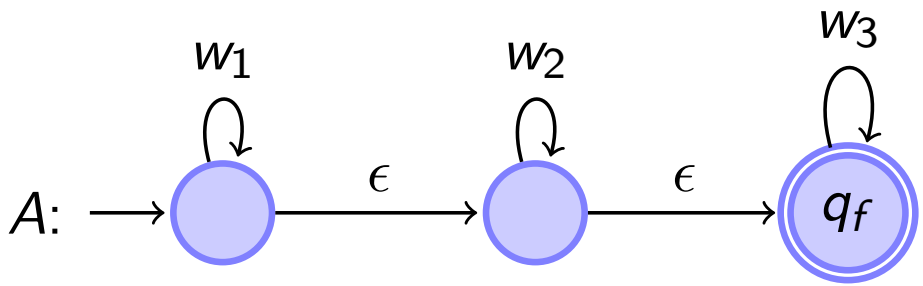

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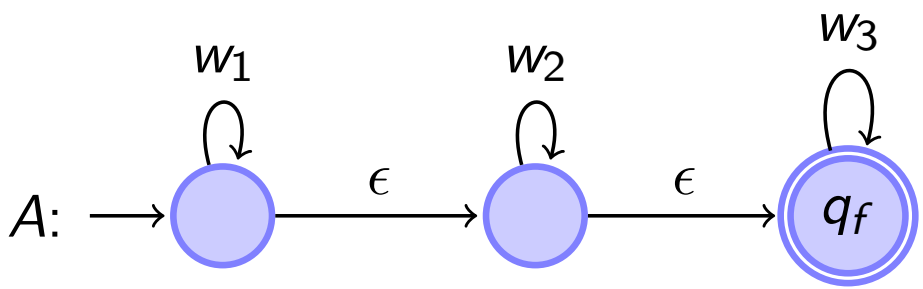

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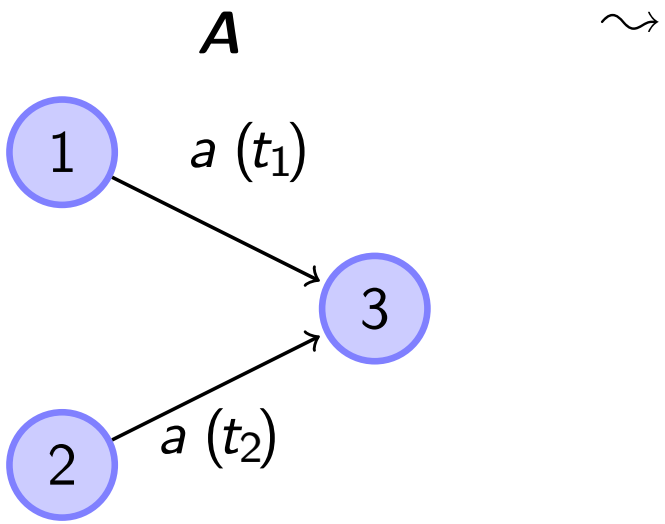

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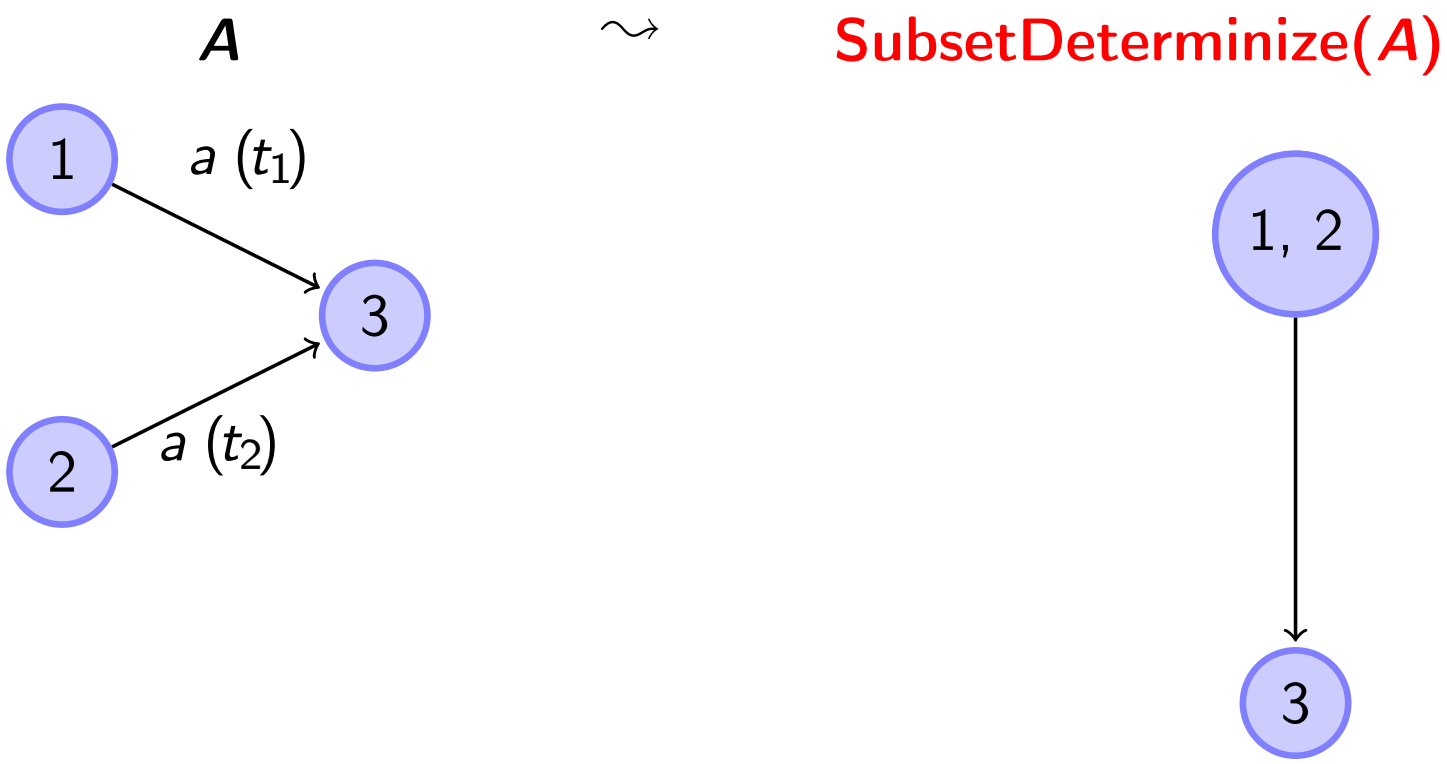

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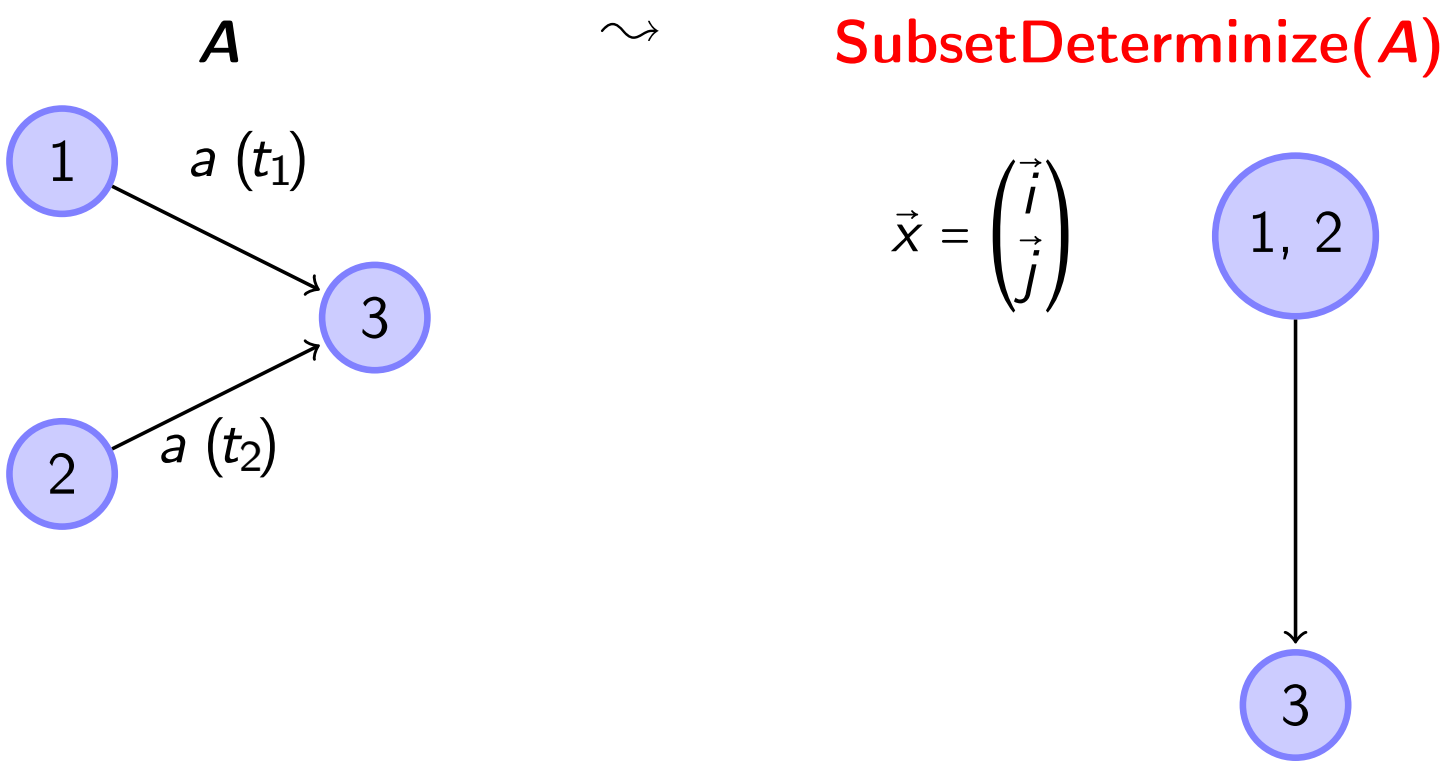

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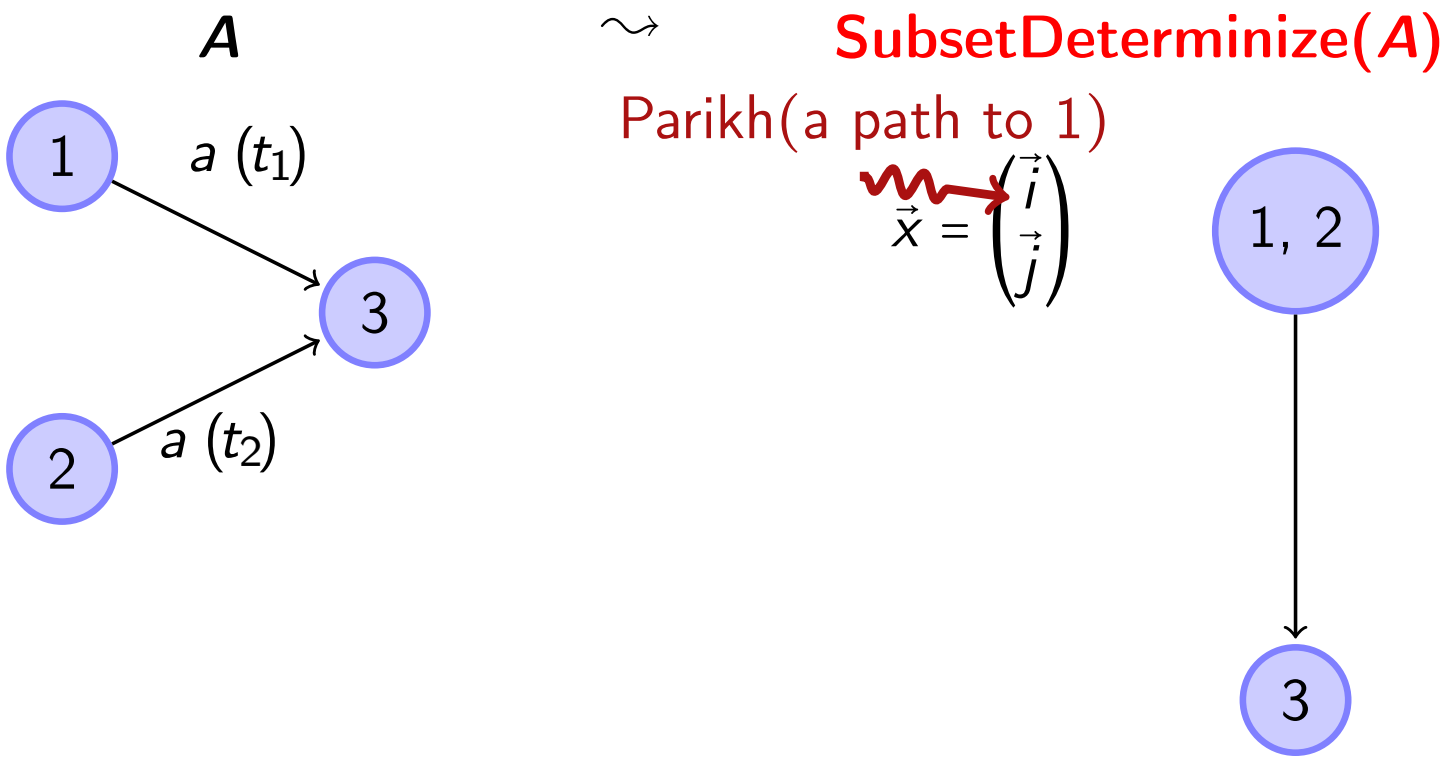

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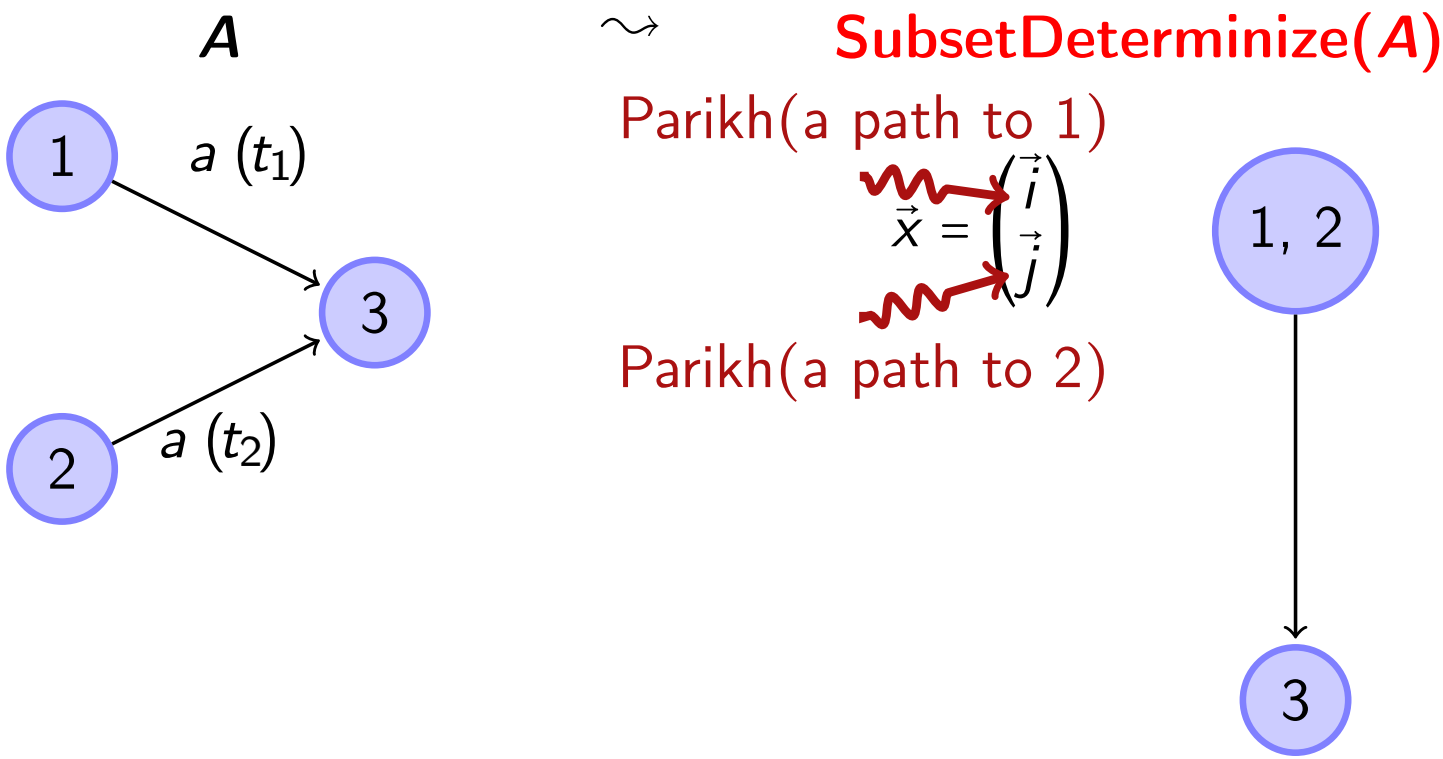

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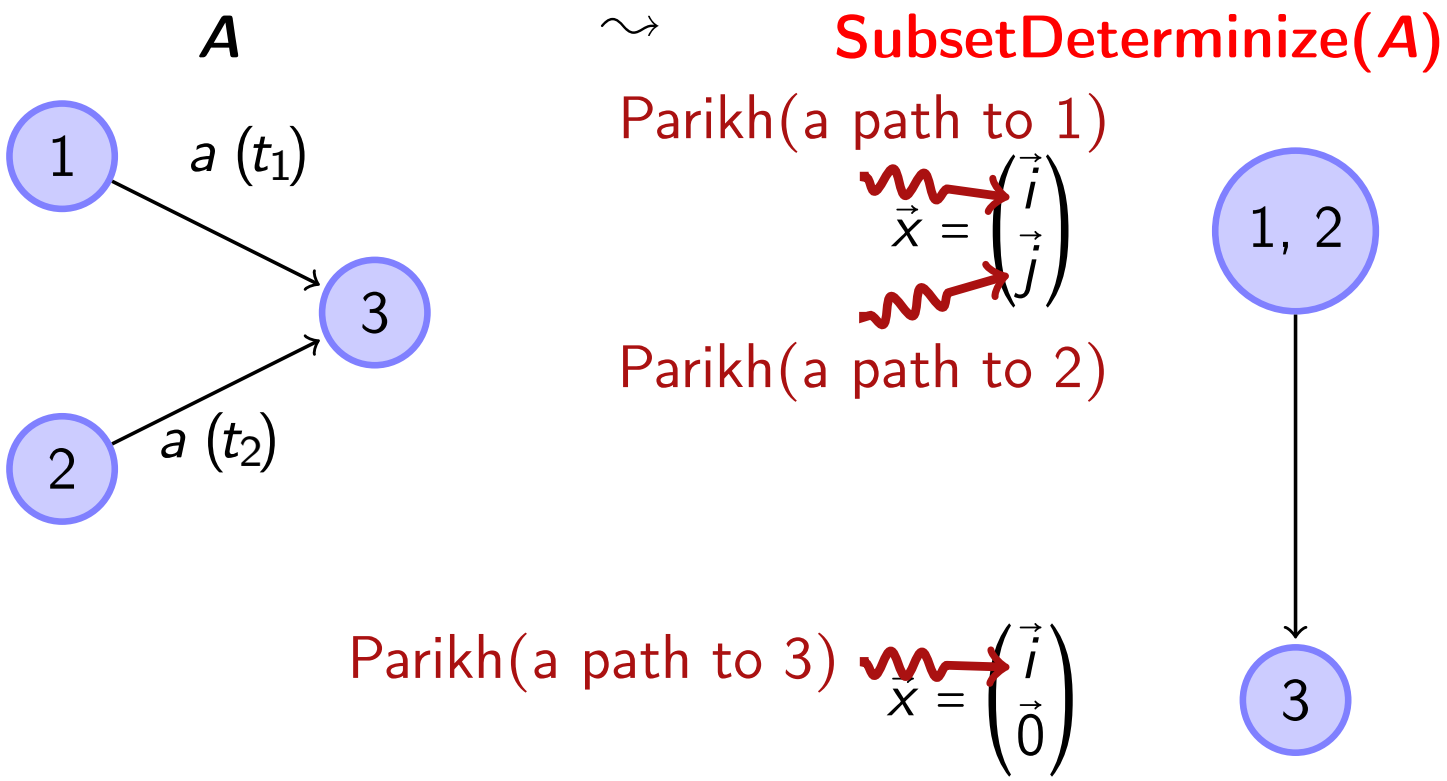

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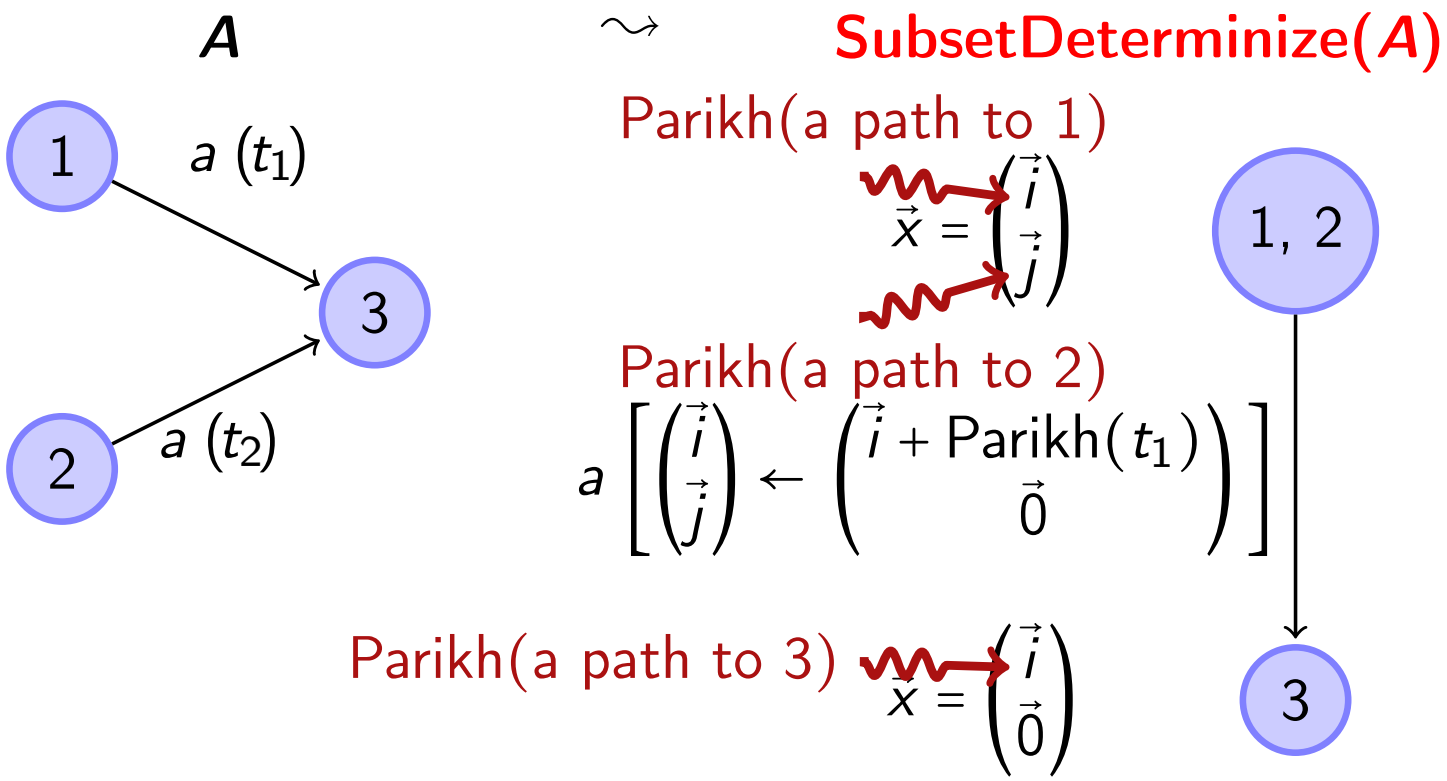

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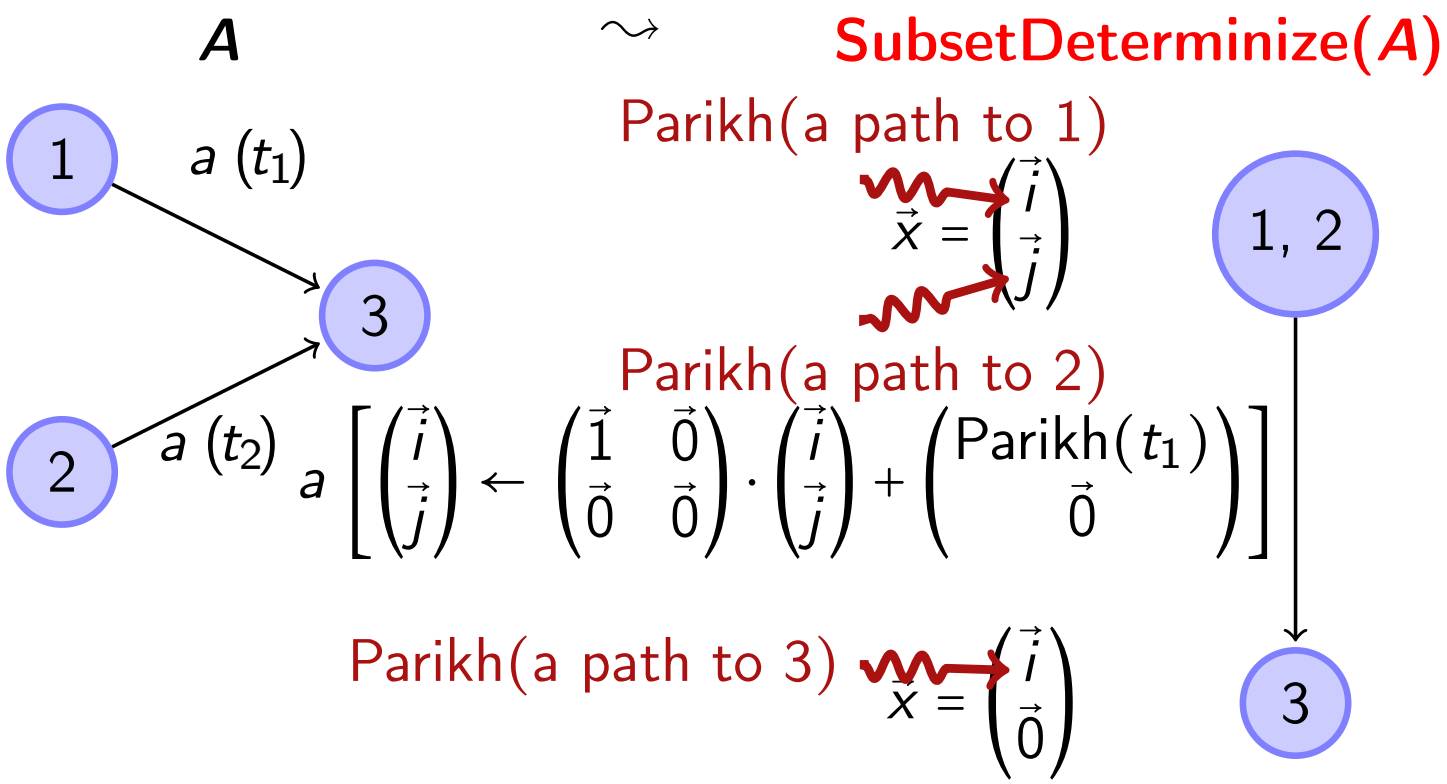

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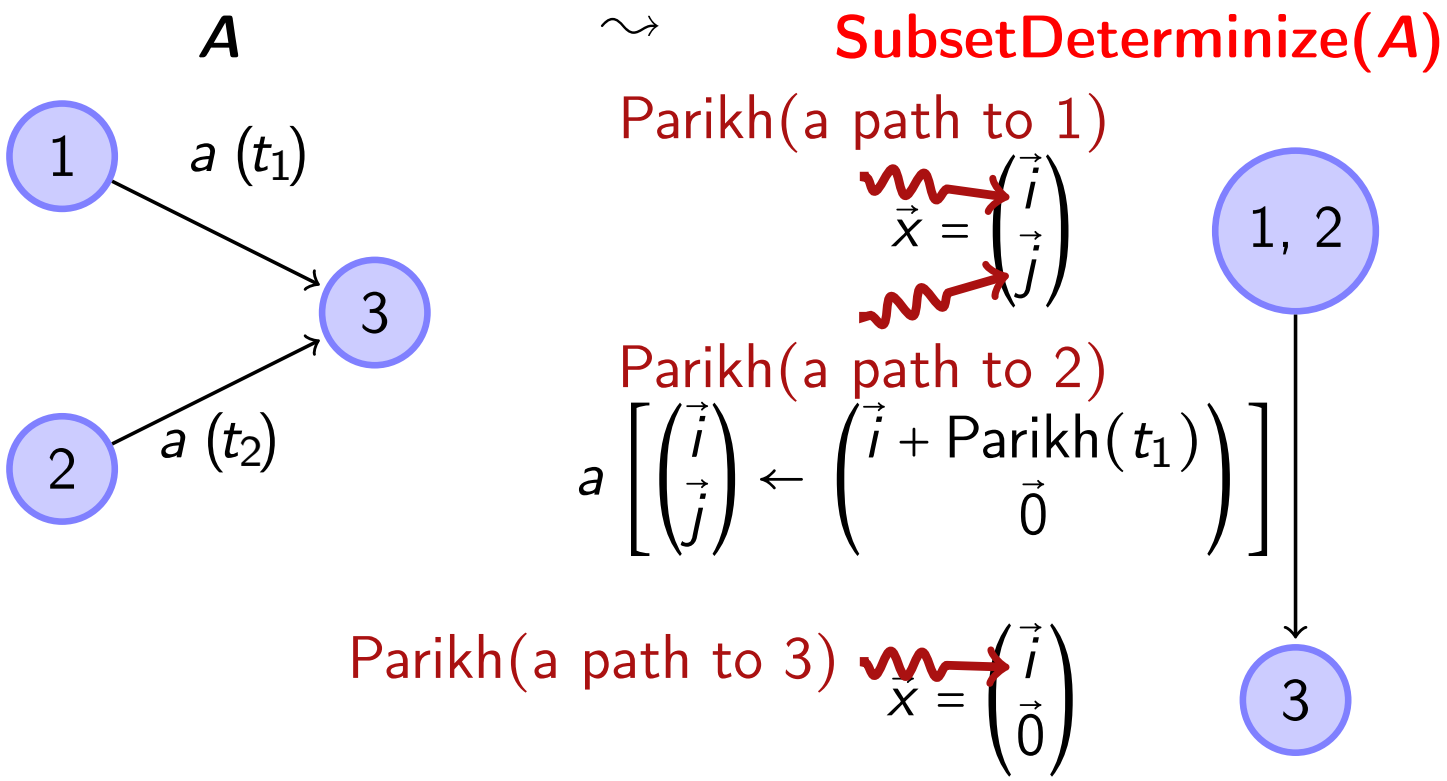

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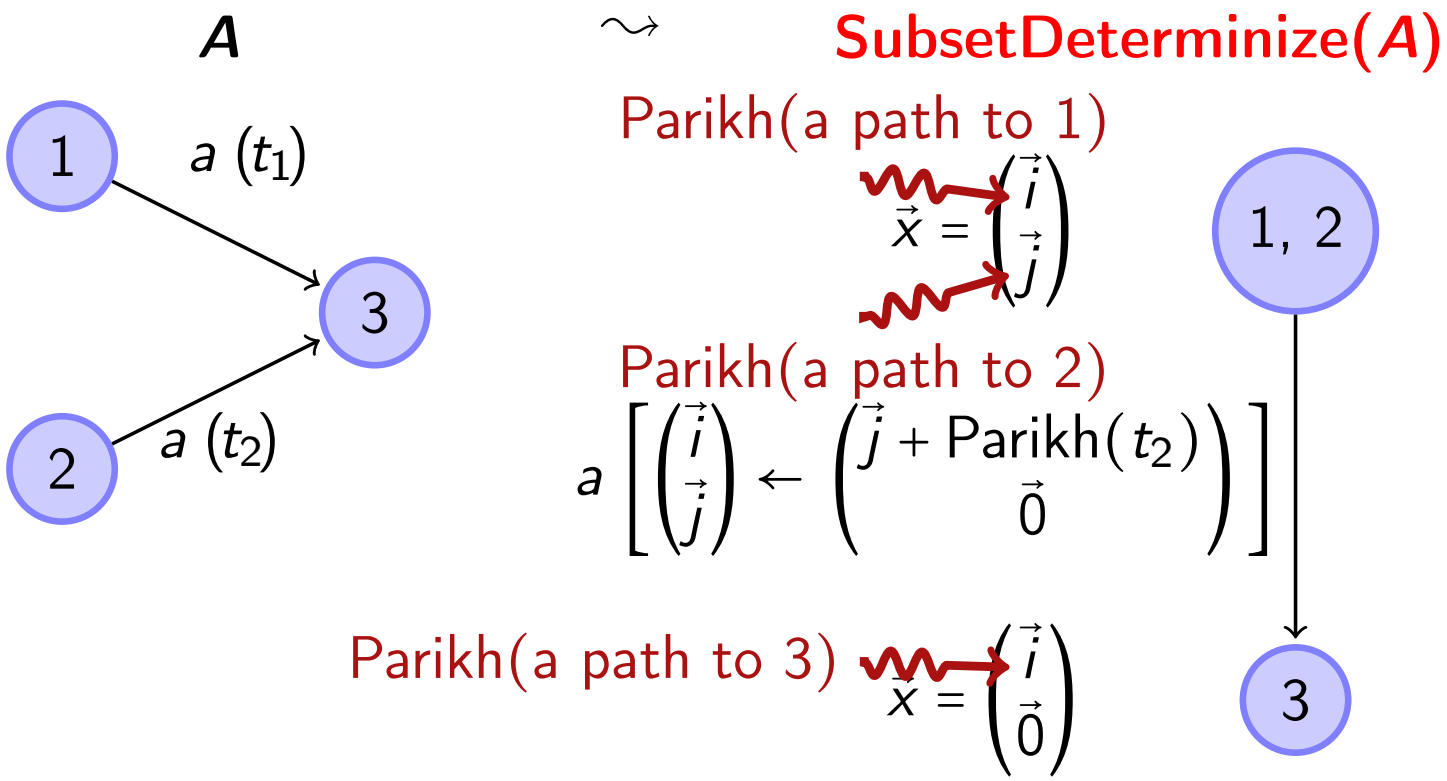

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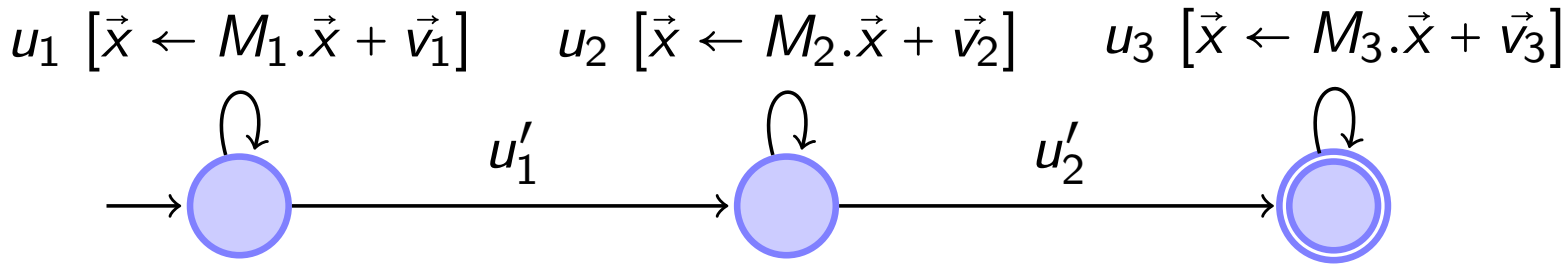

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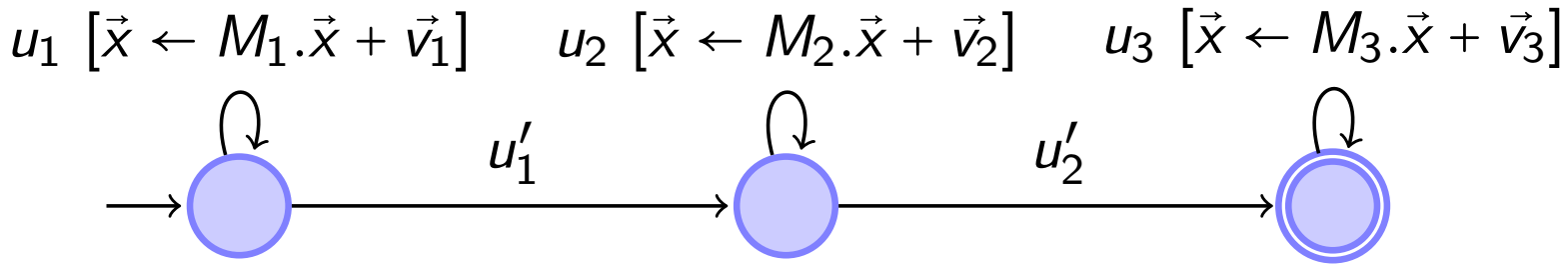

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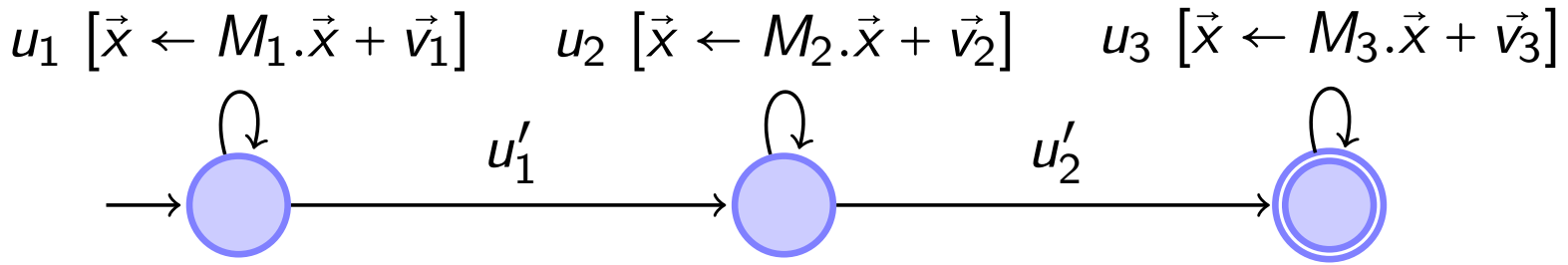

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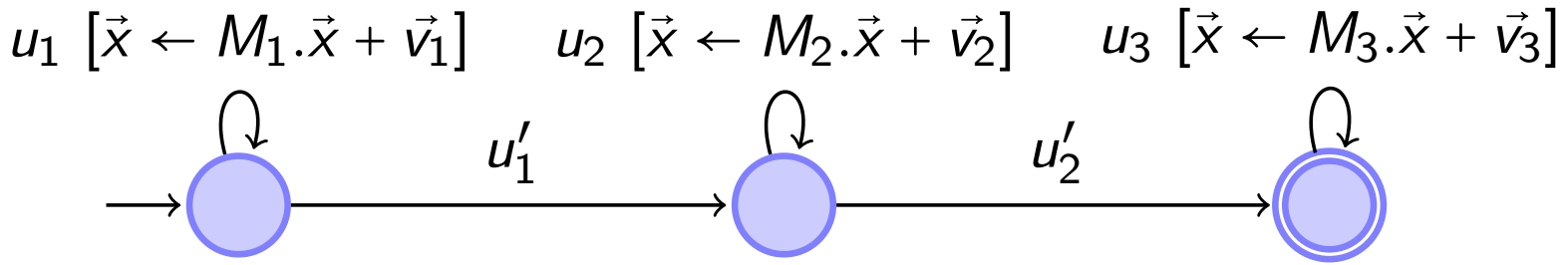

- Property X: For all i, there are p_i, k_i s.t. $M_i^{p_i} = M_i^{p_i+k_i}$
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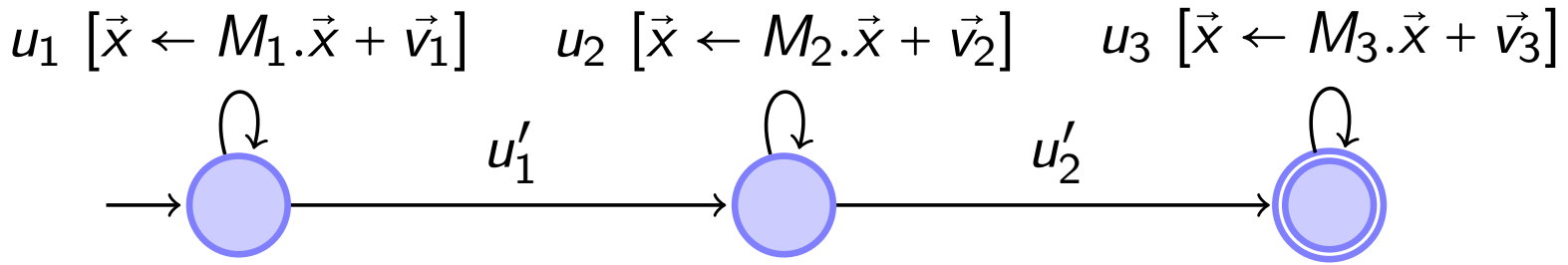

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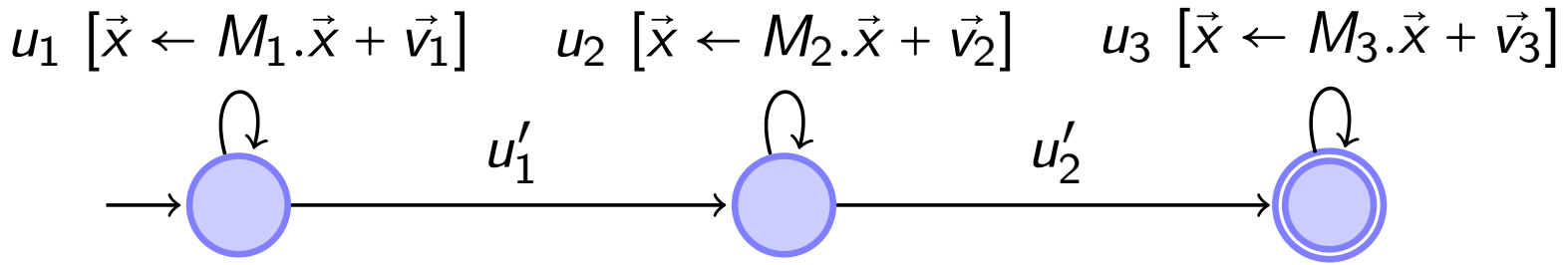

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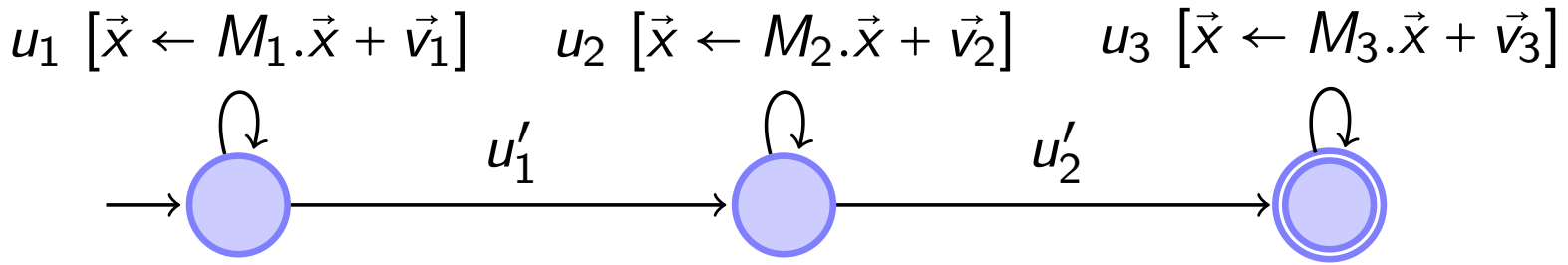

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- Contribution of $(u_2^{k_2})$ *constant*

Outline

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What implications to algebraic/logic characterizations?

Thank you

Result and definitions

BSL ⊆ DetPA

BSL ⊆ DetAPA with some property X

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